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High end complexity

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Despite the general recognition of complexity as an important concept and decades of work, very little progress has been made in the attempt to define complexity. It is suggested that this is due to the fact that the definition of complex behaviour is itself complex, forming a scale from the simple to the more and more complex. Those systems at the high end of the scale are not at present well modelled, and reasons why this might be the case are presented. The possibility that quantum theories may be able to model such high end complexity is investigated.

Keywords: high end complexity, contextuality, objectivity, subjectivity, separability, quantum models of complexity

1 What is complexity?

Despite the general recognition of complexity as an important concept and decades of work in fields such as cybernetics, holism, systems theory, synergetics, *etc.* we are still a long way from a consensus about what such a concept actually entails, let alone a definition (Edmonds, 1999; Horgan, 1995). The list of attempts to define complexity appears to be almost never-ending, and yet very little progress has been made. This work will adopt a different perspective, namely, it will suggest some reasons for this general failure, and propose some new directions in which fruitful research might be conducted. In particular, the hypothesis will be made that the reason for the failure to find one true definition of complexity is that the concept of complexity is in itself complex; there is every reason to suppose that a single objective definition does not exist.

In order to approach the concept of complexity, we must first understand the more easily defined notion of simplicity. Traditional forms of analysis have generally revolved around simple systems which almost by definition may be straightforwardly separated from the ‘environment’, or that part of the system which is deemed irrelevant. This dramatically simplifies the process of modelling the system, as it allows for the implementation of a reductive analysis. This technique of breaking an apparently complex problem into smaller, more manageable pieces, *reduction* and then combining the solutions obtained from these smaller problems into a larger solution which represents the original system, *synthesis* has been remarkably successful in science. In fact, the common conception of science is almost synonymous with reduction, leading to the often mentioned distinction between the so-called ‘hard’ and ‘soft’ sciences. It might be said that a field falls into the hard or soft categorisation depending upon whether or not it is amenable to such an analysis (Rosen, 1991). Thus, fields such as sociology, anthropology, biology *etc.* which examine systems that are not so amenable to the reductive approach are often classified as soft, somehow less scientific, than those such as physics and chemistry, a distinction which is not useful and can lead to some rather controversial debates.[1]

The delineation made by reductive analysis, between system and environment is reasonable if the two are well separated, *i.e.* do not interact in a way that significantly affects the dynamics of the system. Such

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[1]Consider for example the Sokal affair, where the physicist Alan Sokal wrote, and managed to publish an essentially nonsensical article in the field of social sciences (Sokal, 1996b,a; Dawkins, 1998). It might be asked whether similar situations do not (admittedly unintentionally) arise in the more traditional hard sciences; the process of peer review is becoming more and more difficult to fairly implement as the field of knowledge fragments and expands.

a clear separation between system and environment was easy to find in classical physical systems, such as mechanical and thermodynamic ones, where this technique yielded a rich set of very accurate results. However, this separation has never been quite so straightforward for all systems, which often leads to their designation as *complex*.

For example, O'Neill *et al.* understand an ecosystem as something that cannot simply be synthesised from a number of components, but must be analysed in addition to those components:

> *Ecosystems are not simply spatially disjunct groupings of taxa* (e.g., *the plant community). Ecosystems cannot be arbitrarily assigned to a preconceived spatiotemporal framework* (e.g., *the climatic climax). Ecosystems cannot be conceptualised simply as functional entities or simply as collections of species.*
> *Instead, ecosystems must be viewed as systems in their own right.* O'Neill et al. (1986, p37)

This suggests that there is a very real sense in which ecosystems are not separable in the reductive manner, and could therefore be identified as complex.

It is often very difficult to resolve biological systems into components that are independent of one another as they generally exhibit a strong dependence upon environmental conditions (both biotic and abiotic). Often, the traditional scientific aim of separating such systems from their surrounds is impossible. Consider for example the concepts of genotype and phenotype that commonly arise in biology. The genotype of an organism is its genetic content, (*e.g.* its DNA for living organisms, a Bit sequence for Artificial creatures), while an individuals phenotype arises from the interaction of the genotype with an environment during the process of development. This process of interaction between the two can lead to a number of interesting effects, consider for example:

(i) Phenotypic Plasticity, where organisms with the same genotype may, if placed in a different environment, reveal significantly different phenotypes, to the extent that they may even be identified as different species. Such phenotypic plasticity may be realised as alternative morphologies, physiological states, and even behaviour, in response to differing environmental conditions (West-Eberhard, 1989). Thus, evolutionarily important characters do not have to be 'genetic' (immune to environmental effects) reinforcing an often cited (but perhaps not truly recognised) fact that the phenotype is a product of *interaction* between the genotype *and* the environment. Phenotypic plasticity forces us to accept that the phenotype depends in a very strong way upon the environment that surrounds an organism; a different environment can result in a vastly different organism.

(ii) Fitness is not a characteristic that can be ascribed solely to an organism; it must be considered within an environment. *Fitness landscapes* map the fitness of the phenotypes of different organisms. It is well accepted that fitness landscapes can change quite dramatically depending upon the environment in which an organism is found (Brandon, 1990; Maynard Smith, 1993). For example, while it is beneficial that a person's skin colour be darker in colour in regions of high UV exposure, and consequent risk of melanoma, it may be less beneficial to have the same skin tone in less sun-exposed regions where skin pigmentation reduces the efficiency of vitamin D production (which is light-dependent). Also, fitness is not just affected by abiotic factors; the fitness of a species of bird that lays its eggs in a cliff can be quite profoundly changed if there is a species of lizard in the same vicinity that is capable of climbing to the nest.

(iii) Cloning illustrates a number of key systemic interdependencies in biological systems. Firstly, cloning dramatically illustrates the fact that DNA alone will not replicate or form an organism, it must be placed in the context of a cell. Also, the DNA cannot be placed in any cell, it must be placed in an appropriate, as well as viable cell in order to begin replication. Thus, it is not possible to separate genetic content from its surrounds and retain any meaningful sense of a functional system; DNA is contextually dependent upon not just the external environment, but also the cell in which it finds itself, which may be considered a level of environment in itself.

While such problems of separation abound in biology, they are not uncommon in other fields. Many of the environmental problems besetting the modern world result from an inappropriate separation of the economic system from the global environment. For example, given the necessarily finite nature of the world's resources, we might ask how indefinite growth could even be contemplated as a reasonable economic aim. Social theorists and anthropologists are often beset by problems of framework and subjectivity; how they look at some social system often determines the results that are obtained (Mruck et al., 2002). Indeed, the appearance of deconstructive and postmodern arguments might be attributed to the general over application of reductive techniques (Cilliers, 1998; Kitto, 2006). Even that bastion of reductive analysis, physics, is beset by such problems in the analysis of quantum systems; the simplest quantum system must be subjected to *measurements* before its status can be reported, at which point the state of the measurement apparatus becomes crucially important.

How does the inability to consistently separate a system from its environment relate to other definitions of complexity? It is common to identify complex behaviour with models such as Power Laws, Fractals and Bifurcations (Schroeder, 1999), The Renormalisation Group (Kadanoff, 1966; Wilson, 1975), Self-Organised criticality (Bak, 1996), Randomness (Kolmogorov, 1965; Li and Vitányi, 1993; Chaitin, 1990), Catastrophe Theory (Thom, 1975), Dissipative Theory (Prigogine, 1970), Synergetics (Haken, 1988) *etc.*,[1] however, it is likely that such simple identifications are premature. This is because it is possible to identify two broad groups of researchers investigating complex systems. The majority view tends to equate the above systems with complex behaviour and an implicit claim is usually made that further analysis of apparently more complex systems will yield to the same or similar analysis. However, there is a second smaller group of researchers who appear to claim that there is some element missing from such a treatment of complex systems, that complexity might consist of more than just randomness, dissipation and self-organisation (Rosen, 1991; Casti, 1986; Edmonds, 1999; Cilliers, 1998; Pattee, 2001; Corning, 2002). This second group of researchers do not generally consider the systems described by the above theories as truly complex, or at least not *very* complex. Systems amenable to the above techniques are sometimes identified as *complicated* rather than complex (Rosen, 2000), a distinction that is possibly worth making, but our terminology cannot realistically be changed at this point in time given the weight of already existing literature. In any case, such a polar distinction is perhaps not the best resolution to this problem; it appears reasonable to suggest that there is a scale of complex behaviour, with simple behaviour gradually giving way to the more and more complex.

This idea will be examined throughout this work. One characteristic of very complex systems, that of contextual dependence upon an environment, shall be identified, and some techniques which are capable of incorporating this behaviour into our modelling will be proposed.

First however, we must discuss an issue that often arises in the attempt to characterise complexity. This stems from a tendency among researchers to associate complexity with observations, that is, with our accounts of a system rather than the system itself. Does the phenomenon of complexity actually exist?

2 The epistemology versus the ontology of complexity

Complexity is often equated in a rather circular manner with emergence; a complex system is one where new, or emergent, behaviour arises which is not obviously apparent in its definition. This sometimes leads to an attack on the very existence of phenomena such as complexity and emergence, where the stance is taken that emergence arises only in our understanding of systems, rather than in the systems themselves:

> *The relationship between novelty and its evaluation can be made explicit by thinking always of some observer that builds a model of a process from a series of measurements. At the level of the intuitive definition of emergence, the observer is that which recognises the "something" and evaluates its "newness". ... The closure of "newness" evaluation pushes the observer inside the system. This requires in turn that intrinsic emergence be defined in terms of the "models" embedded in the observer.* Crutchfield (1994b, pp3–4)

This amounts to the stance that the emergence of complex behaviour happens only relative to our modelling and is not a phenomenon *per se* (Crutchfield, 1994a; Rabinowitz, 2005; Bedau, 1997). In such an understanding, the emergence of complex behaviour is a byproduct of our perception; it is used by us to make sense of processes that would otherwise be too difficult to understand. However, it is quite clear

[1]Perhaps the most valuable complexity resource is the *Hypertext Bibliography of Measures of Complexity*, created by Bruce Edmond's which while no longer maintained by him, and therefore missing references from later than 1997, contains a vast, fully crossreferenced list of articles that cover the many different ideas surrounding complex systems that had been proposed to that date. This, and a number of other resources are available at http://bruce.edmonds.name.

that some form of emergence has taken place during the history of the earth. For example, according to the most generally accepted theories of biological evolution the Earth did not originally house life of any form, but gradually living forms arose, mutated and changed. We now classify them as organisms, species, phylogenies *etc.* While there are some borderline cases of species identification, it is generally agreed that species now exist which did not do so in the past, and this has a noticeable effect not just in our understanding of organisms, but upon the behaviour of the biotic components of the Earth's environment. According to the most generally applied definition of species, interbreeding is not generally allowed and this is in fact observed; new species have most definitely emerged during the biotic history of the Earth.

We can distinguish between two categories of emergence; *epistemological emergence* which allows reference to emergent complexity only with respect to an observer and *ontological emergence* which allows for the actual emergence of new, complex behaviour (Silberstein and McGeever, 1999; Beckermann et al., 1992). The tendency to associate emergence with epistemological emergence alone stems from a number of different factors.

The most naïve approaches towards emergence tend to find that this phenomenon is largely epistemological due to the simplicity of the systems which are examined. For example, the extraction of the thermodynamic gas laws from the more fundamental theory of statistical mechanics is often cited as a case of emergence. With a distinction between epistemological and ontological emergence we start to see that this case is perhaps not as interesting as might be thought. In fact, it is possible (but very difficult) to describe all of the dynamics of gaseous systems with reference to statistical mechanics. The leap to thermodynamics is primarily one of utility; our models are simplified if we make the leap to a higher level of modelling. It is not the case that new behaviour is actually emerging in this example. Such approaches tend to look at formal representations of the system at two levels, and to associate emergence with epistemology alone, claiming that we could understand such systems in terms of the more fundamental theory but for the difficulty of representation. However, many of the systems in which we are currently interested are not so straightforwardly analysed as formal systems, let alone reduced to a lower level theory. For example, while we have a very good understanding of how the process of evolution couples with natural selection to generate the amazing biological complexity that we see around us, formal and computational models of this process tend not to exhibit such variety (Kitto, 2006), which suggests that this process may not be well modelled by formal models, and that a reduction of this process to lower level theories may not indeed be possible. If such a program cannot be realised then emergence will be unlikely to take this simple epistemological form.

More interesting approaches tend to examine the problems inherent in identifying novelty or emergence, pointing out quite correctly that there must be an observer capable of recognising the novelty of the system (Baas, 1994; Crutchfield, 1994b; Löfgren, 2004; Minati and Pessa, 2006). While fundamentally correct, such approaches risk lapsing into an epistemological understanding of complexity and emergence, where such phenomena are understood to occur only in the mind of an observer. However, this problem arises only if an inappropriately narrow understanding of observation is adopted; one that relies upon conscious beings capable of performing observations and then communicating such observations among one another. Observation should be understood in a far broader sense. Again, this can be seen with reference to the process of biological development discussed above. During the course of evolution, new channels of interaction within cells have emerged, and these can be observed by other developmental processes during the actual growth of an organism well before conscious observers were able to record and interpret the event. If observation is limited to a conscious observer actively modelling then such emergent processes will not be modelled by our complex system theories.

3 A complexity scale?

> *Simplicity may have a unified form, but complexity has many varieties. . . . Concrete complex systems spread across a whole spectrum of complexity. For systems on the high-complexity end of the spectrum, such as brains or persons, our current sciences offer catalogues of facts but no comprehensive theory.* Auyang (1998, p9)

Looking at the above biological examples, we might start to form an understanding of the reasons why they have not been well modelled by the reductive approach. Each displays behaviour that might be termed *contextual*. The environment, or context, of the system under study is often as important to the dynamics as the system itself. Thus, these systems are *non-separable* from their context, and the reductive approach is difficult if not impossible to apply if we seek to understand their full dynamics. Such systems cannot be understood using our current simple reductive methodologies.

While simple behaviour is relatively straightforward to define there appears to be a wide range of complex behaviour. It seems possible to identify a complexity scale, which moves from simple systems described by simple theories which make reductive or separable assumptions (*e.g.* the Newtonian understanding of projectile motion), through those theories that describe more complicated behaviour consisting of, for example, a large number of components but which are still essentially separable (*e.g.* statistical mechanics), and then into the broad class of systems often identified as complex. Within this class of complex systems we can place a number of those analytic approaches traditionally associated with complexity, but which apply reductive assumptions, at the low end of the 'complex behaviour' spectrum before we move into the behaviour that is more poorly modelled by such theories. At the high end of such a scale we would expect to see processes such as biological development, the evolution of mind, language and societies *etc.*, processes which exhibit significant contextual dependency and are not well modelled by our current theories. Figure 1 illustrates how such a scale might look at this stage of our understanding.

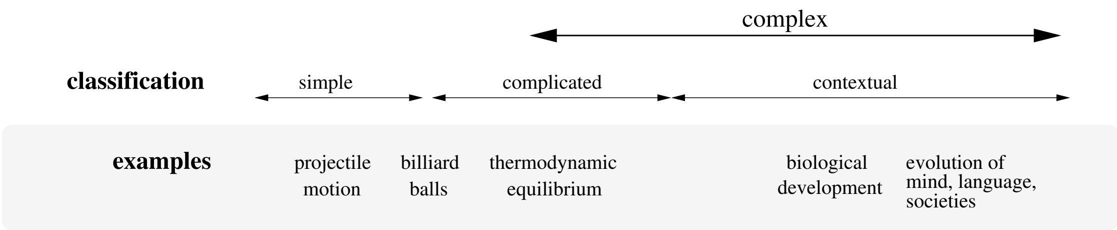

Figure 1. An initial complexity scale. In this framework there are not simple and complex systems, rather all systems are understood as belonging to the scale, but some are 'more complex' than others. Such a scale would move from simple mechanistic systems, the dynamics of which are well understood using current reductive methodologies through to those systems and problems which might be understood as exhibiting very complex behaviour.

Most of the analytic approaches traditionally associated with complexity are still applying largely reductive assumptions and hence they will not be able to model the full dynamics of systems at the high end of such a complexity scale. For example, almost by definition the Renormalization group (RG) approach assumes separability. It would not be possible to move to coarser and coarser grainings if the system of interest were significantly nonseparable or interacting as such, since, in this case important parts of the dynamics would be lost. Historically, the RG was applied only to systems whose dynamics approximated a free field system, and in fact, while it has proved very successful for many such systems, it fails for systems with strong correlations (Delamotte, 2004).

It is interesting at this stage to consider the spread of theories covering such a scale. While systems at the simple end of the spectrum are well understood, often with reference only to a couple of well utilised theories, there are many more theories available for those systems around the middle of the scale (at the low end of the complex system classification) indicating the enormous amount of work being performed here under the guise of Complex Systems Science. However, there are far less theories covering those systems at the higher end of this class. If there is indeed a dramatic difference in the modes of analysis required in order to understand such systems, then it is likely that we will not significantly enhance our understanding by gradually chipping away at the 'easier' systems. Instead we must develop new tools and modes of analysis capable of understanding and describing such behaviour.

Similar scale-type proposals have been made by other authors (Chu et al., 2003; Auyang, 1998), but never with any actual suggestion of how such a scale might look, or indeed affect our analysis. One very real possibility that suggests itself is that such a scale should not be linear, rather that the characterisation of a system as simple or complex will depend upon a number of factors, this is an idea that will be reserved for future work. At present we can note that it is actually very difficult to provide any specific form of scale due to the lack of theories at the high end of the scale; how are we to classify such systems if we have no theoretical understanding of them? It seems likely that before a proper understanding of any possible scale of complexity can be obtained it will be necessary to develop new modes of analysis capable of exploring the behaviour of systems at the high end of the complexity scale; those exhibiting what might be termed *high end complexity.*

3.1 *High end complexity*

The discussion of sections 1 and 2 has developed two main themes:

(i) That not all systems can be separated in a sensible manner from their environment or context, a characteristic which often leads to the identification of the system as complex.
(ii) That the identification of emergence in systems exhibiting complex behaviour generally relies upon some notion of observation or measurement, at least in those cases of emergence that are nontrivial.

Combined, these two ideas suggest that a serious examination of very complex behaviour must somehow be able to incorporate both contextuality and emergence defined with respect to some form of measurement or observation. Systems exhibiting both of these characteristics are generally those that might be claimed to lie at the high end of any complexity scale that we might wish to construct, and we must take account of the way in which the context of the system can affect our understanding of its behaviour. The reliance upon measurement or observation in the identification of emergence can lead to interesting problems when we attempt to construct new models of such systems. For example, we might ask how much our language or description of such systems will affect the results of our analysis. Löfgren (1977) has identified two natural complexity classes, *d-complexity* which is the complexity of producing descriptions, and *i-complexity*, or the complexity of producing interpretations, which start to examine the nonseparability of our models from their language of description. Within the current formulation of high end complexity we can understand this nonseparability; simple systems will be those which can be adequately modelled with very little interpretive complexity *i.e.* the description of such systems requires very little interpretation, whereas those exhibiting contextuality, and indeed observer driven complexity will have much higher interpretive complexity.

Although such an identification is likely to be controversial, we shall adopt the stance that at least one characteristic of systems exhibiting high end complexity is some sort of observer dependence. This is a stronger characteristic than that of contextuality; some contextual behaviour is not observer dependent, however, observer dependence results from the contextuality of some systems *under observation.* We must keep in mind the discussion of section 2; observation is used here in a very broad sense.

A number of researchers have pointed to similar characteristics in systems exhibiting complex behaviour (Pattee, 2001, 1996, 1968; Casti, 1986; Edmonds, 1999; Löfgren, 2004), theories which shall be designated hereon as *observer driven.* One of the most comprehensive such theories is due to Edmonds who has formulated a definition of complexity (Edmonds, 1999) based upon inequivalent descriptions:

> *Complexity is that property of a model which makes it difficult to formulate its overall behaviour in a given language, even when given reasonably complete information about its atomic components and their inter-relations.*
>
> Edmonds (1999, p72)

He defends this definition through a comprehensive examination of a number of alternative ideas surrounding complexity, such as patterns, size measures (including the number of components, size of the rule set, or the size of the minimal description in some language), processing time, or computational complexity, ignorance, variety, surprise, and improbability, the midpoint between order and disorder, logical strength, and irreducibility *etc.* claiming that these are concepts “that are frequently conflated with complexity, but which are, at best, very weak models of it.” p57, (Edmonds, 1999). With an acceptance of a complexity scale, such concepts can be seen to lie somewhere in between simplicity and complexity, explaining the weakness of such models as exemplars of complexity, as well as their more complex nature when compared to simpler reductive models. Indeed, Edmonds himself identifies a number of other theories of complexity as special cases of his definition, notable among these special cases are computational complexity, algorithmic information complexity, and Shannon entropy. As these theories are very often identified with complex systems, the ability of Edmonds definition to incorporate them supports the current proposal of a complexity scale, as the possibility of subsuming 'less complex' behaviour within a larger definition suggests that such phenomena are at the lower end of the scale. In fact, such conceptions of complexity are largely syntactic, they make the reductive assumption that the characterisation of a system as complex can be achieved without reference to the way in which it is observed, or to the way in which a model of such a system necessarily incorporates interpretive structures held by the observer *i.e.* the semantics and pragmatics of models of the system.

Complexity defined with respect to an observer, while not immediately appealing to the majority of researchers in complex systems science does sidestep a number of issues that have plagued the general drive to find a definition of complexity:

(i) Complexity often seems only to make sense with respect to some level of description. For example, coarse graining, or the examination of a system on a larger scale (Auyang, 1998), often turns an apparently complex system into a far simpler one:

> *Depending on the spatiotemporal scale or window through which one is viewing the world, a forest stand may appear (1) as a dynamic entity in its own right, (2) as a constant (i.e., nondynamic) background within which an organism operates, or (3) as inconsequential noise in major geomorphological processes. Thus, it becomes impossible to designate* the *components of* the *ecosystem. The designations will change as the spatiotemporal scale changes.* O'Neill et al. (1986, p83)

(ii) As the above quotation suggests, the time frame over which a system is examined can profoundly influence its classification as complex or simple. Organisation is sometimes even identified as *resulting* from this difference in processing rate (Simon, 1996; O'Neill et al., 1986; Salthe, 1985).

This problem has an added difficulty that arises when we consider our current modelling paradigm. It is standard practice to model systems using differential equations, but these generally only allow for constant time intervals (Conrad, 1983). The behaviour exhibited by complex systems often occurs on a range of faster and slower timescales, and this is often lost in a model based upon differential equations. This is a particularly relevant objection when we consider the interesting behaviour that can be generated by examining systems using differing timescales, such as was demonstrated by Turing (1952).

In addition to this problem of modelling, many researchers have emphasised the way in which there appears to be an anticorrelation between the timescale over which an interaction occurs, and it's apparent strength (O'Neill et al., 1986; Salthe, 1985), and such effects should be incorporated into our modelling. Consider for example the dynamics of an ecosystem, which may evolve over a timescale of centuries to millennia, where perturbations such as an increase in carbon dioxide may take 100 or more years to eventuate in changes that impact upon single organisms, which interact on a much faster timescale, and often much more strongly. This can often lead to interesting paradoxes in social reactions: a murder is a very strong interaction between two people over a potentially very short timescale, which often draws a strong reaction from a community, but the gradual warming of the earth, which has the potential to kill far more organisms albeit over a far longer timescale, does not appear to draw the same amount of community ire, at least for now. Generally an interaction over an ecosystem is far slower, and on the same timescale as organismic interactions, far weaker (something that appears to have been somewhat cynically recognised by our politicians). A similar pattern repeats as we compare the interactions between cells in the organisms to interactions between the organisms themselves *etc.*

(iii) As will be discussed in section 4.1, it is often difficult to identify the objects that should be taken as fundamental in the modelling of complex systems. Depending upon the behaviour of interest it is often necessary to make use of a different set of primitive objects as well as dynamical equations (Auyang, 1998).

Observer driven definitions of complex behaviour sidestep these issues because different observers will see different objects as fundamental depending upon their spatiotemporal level of interest, and the dynamics that they extract from a model of a system will similarly be dependent upon their viewpoint. However, it is necessary that some sort of systematic connection between these different descriptions be possible, even if the connection does not make two different models equivalent. Without such a connection, we risk lapsing into the epistemological conception of complexity discussed in section 2 where complexity can be defined only with reference to an observer, *i.e.* subjectively.

Despite the above advantages, the observer driven approach has not tended to attract the general attention that it deserves, especially in the traditionally more reductive fields such as physics. We might attribute this lack of recognition to the perceived negative implications of such an approach. In the general excitement created by the new techniques of 'complex analysis' there was a feeling of optimism that it would be possible to describe increasingly complex systems without a substantial rethink of the dominant modelling methodology. However, as the preceding sections have suggested, it is becoming apparent that this is not likely to be possible.

3.2 *A proposed complexity scale*

With the notions of contextuality, high end complexity and observer driven theories, now more fully fleshed out we are in a position to return to the complexity scale. In addition to the classification of systems that was begun in figure 1 we might start to rank our theories, tools and conceptions in order of increasing complexity. Indeed, such a procedure will assist in our attempt to classify the systems they describe, since those systems that can be described by a simpler conception of complexity will presumably be less complex themselves. As discussed above, many of the techniques used in analysing complex behaviour (such as the RG approach) are based upon separable assumptions and hence should not be considered to lie in the realm of high end complexity, however these theories are more complex than say, Newtonian mechanics, and can probably be seen to lie somewhere in the complicated category (see figure 2). Also, information theoretic conceptions generally ignore contextual factors, assuming separability right at the outset which means that they are probably not measures of any use when it comes to the analysis of nonseparable systems. As has been discussed earlier they are generally only measures of behaviour at the fairly low end of the complex scale. Indeed, the general ignorance about structural and environmental information in these approaches to complexity suggests that they are less complex again than say, RG approaches which at least have analytic techniques built in for determining whether a system might be considered as separable. For these reasons such theories are listed as less complex again. Approaches such as network theory have the ability to incorporate more contextual information into their models. For this reason food webs (Pimm, 2002) are seen as more complex than population dynamics, with genetic regulatory networks (Barabási and Oltvar, 2004) seen as more complex again due to the substantial increase in the number of components necessary to describe them, as well as their associated interactions.[1] Postmodernism is included in this scale due to an interpretation of this theory as an effect resulting from an overapplication of the reductive technique to very complex systems (Kitto, 2006), a result that will be discussed in more detail elsewhere. Evolutionary approaches are significant as one of the few that are capable of modelling complex emergent behaviour, and they are therefore depicted at the high end of the scale. However, simple evolutionary models such as are often utilised in ALife do not generally achieve high end complexity due to their inherent simplicity (Kitto, 2005, 2006). Thus, while evolution is a theory of high end complexity, it does not necessarily generate such behaviour; the model itself must be sufficiently complex.

Figure 2 is a preliminary suggestion about how the ordering of such theories might appear, at least when collapsed along the axis of contextual behaviour. This proposal will not be justified in any more detail here; it is not something that can be justified without an extensive debate within the complex systems community, however, it seems likely that such a scale can be constructed and that the concept will be useful in the analysis of complexity.

Even at this preliminary stage, it is possible to make a number of observations with reference to this proposed complexity scale. For example, while it is likely that those systems exhibiting high end complexity will be observer driven, not all contextual theories will have such a characteristic, that is, it will not always

[1] If food webs generally considered more components, such as bacteria, parasites and viruses in their models then they would probably approach the complexity of genetic regulatory networks. In fact this points to the simpler nature of food webs; we generally do not need to include these extra components in the models of interest and hence their effects are separable from the model. This also emphasises the subjectivity of classification which can creep into this form of analysis; a different analysis may always be far more complex than is the norm.

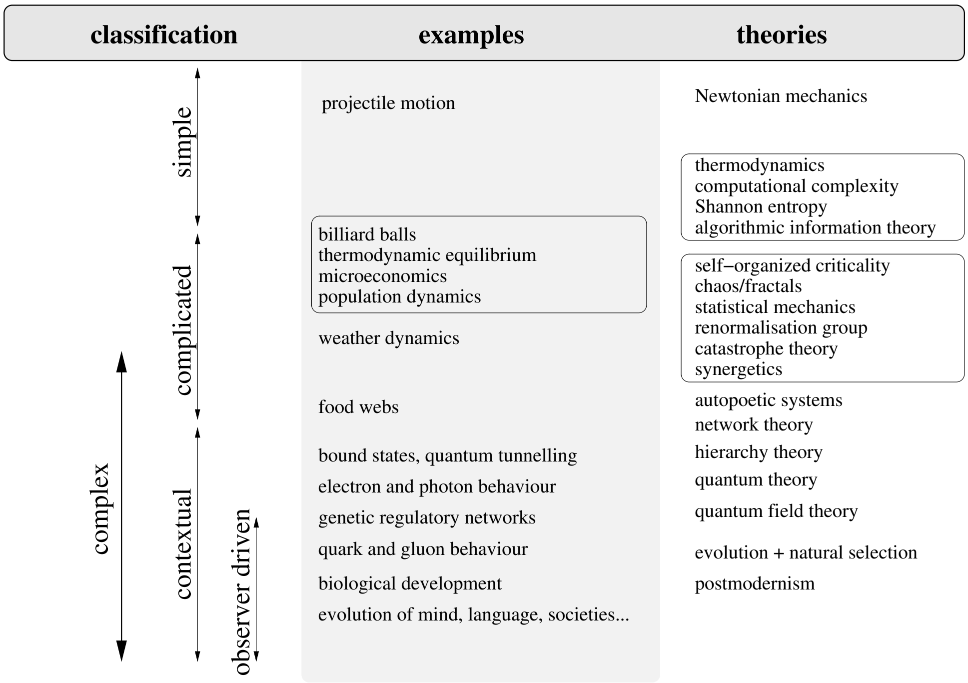

Figure 2. A proposed complexity scale. In this framework there are not simple and complex systems, rather all systems are understood as belonging to the scale, but some are 'more complex' than others. The complexity of systems and their associated theories increases as we move down the scale. Theories and examples that are boxed in this figure are considered to have a similar level of complexity. Although this scale is drawn as one dimensional it is unlikely that a final classification will take such a form, a number of characteristics are likely to be important in the identification of a system as simple, complex *etc.*

be necessary to describe a contextual system using more than one model or language. Hence, contextual systems are not necessarily very complex, and they might be understood to fall over a broad range of the complexity scale.

Thus it appears that there are differing levels of observer dependence within complex systems:

(i) Contextuality refers to a dependency of a system upon some aspect of its environment. That is, a dependency upon anything not directly associated with the system itself. Such dependencies might include measuring apparatus, timescales, background noise, *etc.*

(ii) Observer Dependence is a stronger term, encompassing contextuality but requiring in addition that the contextual dependency occur with respect to measurement or observation; a system with an observational dependence will be contextual, but not all contextual systems will exhibit a direct observer dependence. Systems exhibiting observer dependence cannot be fully described by a single model; in incorporating the choices of an observer it is necessary to adopt at least a two tiered approach to the description of the system. However, there is every reason to suppose that these observer dependencies can be communicated in a meaningful manner. Thus we can hope to approach an understanding of observer driven complexity.

(iii) Subjectivity is a stronger term again. A subjective description of a system cannot be communicated in a meaningful sense. It is likely that while subjective descriptions are important (consider for example the importance of personal responses to visual art, music and literature) they are beyond the purview of science.

Thus, a system could be dependent on an observer in a well defined manner, in which case it would not be fair to identify it as subjective. If it was possible to formalise this dependency in some sense, then the description of the observer dependency could itself be communicated and a sense of objective description saved, even if a contextual system were exhibiting strong dependencies upon the choices made by an observer interacting with it.

Returning to the proposed scale, we note that there appear to be far more techniques developed for the low end of the proposed complex systems scale, with an apparent lack of theories and techniques that can be applied to those systems falling in the higher end of the scale. Quantum theories have been added to the high end of the complexity scale in anticipation of section 5, but without these theories there are almost no theories available for the analysis of systems at the high end of the scale. Finally, with reference to the Edmonds definition of complexity, we see that complexity is itself a complex concept. Many different models and tools are required in order to understand such behaviour; no one language of description is sufficient. This raises an interesting dilemma; if complexity can be defined as a difficulty to formulate the behaviour of a system in the language of one model alone, then it is remarkable that there are so few fundamentally different models of complex systems, especially at the high end of the scale. We require new models which emphasise the contextuality of complexity.

4 Modelling complexity

The complex nature of complexity has some interesting ramifications when it comes to the modelling of high end complexity. Existing models of complexity have tended to apply at the lower ends of the complexity scale, and while they are relatively successful in this area, there are indications that this will not be the case as we attempt to construct models aimed at understanding the phenomena that might be seen to lie on the higher end of the scale. This section will discuss some of the issues involved in modelling such phenomena before the next section turns to some existing formalisms that show some signs of yielding more results in the modelling of high end complexity.

4.1 *The barrier of objects*

Our understanding of the world is *object based*; we see chairs, aeroplanes, trees, dogs *etc.* and form theories, models and predictions about their behaviour. We are often even right in our predictions. It is this object based methodology which has tended to form the foundation of our reductive understanding of the world. The general idea has been that if it is possible to understand an aeroplane as composed of engines, seats wings, wiring *etc.* then it seems reasonable to expect that other aspects of the world could be understood in a similar manner. However, there are reasons to suspect that such object based models cannot be as effectively used for all physical systems. Specifically, if we assume an object based approach at the outset of our modelling, then it becomes impossible to generate the open ended emergence of new behaviour; the assumption of objects at a fundamental level of the model generally fixes the available channels of interaction within that model and they get 'used up', rather than generated, as the model evolves (Kitto, 2005, 2006).

Even without admitting that phenomena such as complexity and contextuality are disrupting our theories we can find other reasons why object based methodologies are coming to the end of their general validity. Consider for example the infinite regress that is being faced by physicists attempting to define the fundamental objects of the Universe. Over a period of centuries our understanding of the world in terms of the substances we see around us has been refined, first to the atomic level, then atoms themselves were explained in terms of nuclei and electrons, and nuclei are now understood as emergent structures formed from quarks and gluons (Marshak, 1993). At present, a number of models of these 'fundamental objects' have been proposed, including strings (Sen, 1998), branes (Neeman and Eizenberg, 1995), preons (Marshak, 1993) and loops (Rovelli, 1998), but we might ask even at this point what the constituents of these new objects will be. Something is obviously wrong with our methodology. This problem becomes particularly evident when we consider the nature of quarks. Considered simplistically, a nucleon is composed of three quarks undergoing a very complicated set of interactions in what is termed a colour singlet state. Although quarks appear to be obvious contenders for the role of parts of a nucleon, the *colour confinement hypothesis* (Marshak, 1993), suggests that they do not make sense individually, and no individual quarks have ever been discovered in nature. Instead they occur in certain allowed 'colour neutral' combinations. There is therefore a sense in which, while a quark is a reasonable modelling tool, it is a far more complex phenomenon individually than a nucleon. Thus, while it makes sense to talk about an electron and a proton, and experiments can be performed with these objects, the same cannot be said about quarks; as objects, quarks exist tenuously at best. It is most likely that they are more real in our modelling then they are in reality.

This example should cause us to pause in our relentless pursuit of objects. However, it presents its own difficulty; how are we to construct non-object based models? Our entire modelling apparatus, indeed our very language, consists of identifying objects and their interactions. Our mathematics mirrors this, if object based models are insufficient in an exploration of reality then we are left in something of a dilemma.

This problem has been recognised in the field of physics (well-known examples include Bohm (1981), and Wheeler (1992)), indeed the development of the field concept can be considered as an attempt to sidestep some of the issues associated with this barrier; as a modelling tool that is potentially spread over an infinite 'space' it is difficult to conceive of a field as an object in the traditional sense. Indeed fields do go some way to replacing the object driven methodology of traditional analysis, especially upon quantization when 'particles' which are in fact very complex fields can be understood as themselves composed of 'particles' ad infinitum. Despite this, the full power of the field concept has yet to be widely acknowledged, perhaps because of the way in which practicing researchers continue to refer to 'particles' undergoing 'interactions' with other 'particles', a concept that is at best nonsensical when one considers the modern mathematical formulation of Quantum Field Theory (QFT). The use of the field concept in the modelling of high end complexity will be discussed in more detail in section 5.2, but we shall briefly discuss some other interesting proposals here.

(i) Bohm's implicate and explicate order (Bohm, 1981) is a well-known attempt to overcome these problems of modelling. Bohm described the many apparently separate objects and events of the external world as *explicate*; according to him they are merely reflections of the whole or *implicate* order. Bohm assigned primacy to an understanding of reality as an undivided whole, and the implicate order inherent within that whole, rather than to the more traditional reductive parts of this whole, such as particles, quantum states, fields *etc.* In such a framework, objects are necessarily explicate and should not be considered fundamental in our modelling. Interestingly, Bohm's suggestion as to how the implicate order should be understood makes use of a formalism that is essentially quantum mechanical in its structure. There are reasons to believe that such an approach is quite likely to succeed, and some of these will be discussed in the following sections.

(ii) Löfgren's attempt to understand the role that our language plays in our attempts to model systems (Löfgren, 1977; Löfgren, 1996; Löfgren, 2004) is another case where the concept of an object separable from our modelling comes into question. Löfgren examines the linguistic predicament associated with attempting to understand a system without reference to the language that is used in its modelling. In such an approach the object under study cannot be considered without a proper understanding of the language or framework in which it is described and that description interpreted.

(iii) Pattee (2001, 1996, 1968) attempts to understand the way in which the *object*, the system under study, is separated from the *subject*, the thing that is interacting with (*e.g.* measuring) the object. Pattee claims that an epistemic cut is necessary, and is necessarily outside of the bounds of standard reductive analysis in the attempt to understand any system that involves a situation of measurement or control. In the attempt to model such a cut, it is likely that our notion of the object will have to be significantly extended, and the standard object driven methodology of science changed.

(iv) Another example of scepticism surrounding object-based methodologies arises in some of the more sophisticated approaches to emergence. For example, the Baas notion of emergence as a property that is noncomputational, and his attempt to define emergence with reference to a process of observation Baas (1994) suggests that again the conception of objects as immutable objects that we measure, rather than interact with is coming under attack within the scientific community.

Interestingly, all of the above approaches appear to be questioning the traditional scientific ideal of objectivity; that we can understand reality without reference to the methods that we are using to examine some system of interest. This ideal can be seen to lead to another of the barriers currently thwarting our attempts to model systems exhibiting high end complexity.

4.2 *The barrier of objectivity*

Closely associated with the barrier of objects is the barrier of objectivity. As we have seen, it is common to assume in our reductive modelling that systems can be consistently dissociated from their environment, or context, but as our understanding of natural systems progresses we are finding examples where this is not necessarily the case. There are many systems that we wish to understand which are apparently displaying contextual behaviour and hence cannot be cleanly separated from their surroundings. These surroundings might include the experimental arrangement itself, factors external to the apparatus (*i.e.* the environment surrounding the system and experimental apparatus), the history of other experiments performed upon the system, *etc.*. Such contextuality is often perceived as negative, leading to the loss of realism, but this is a far stronger claim than is justified. However, there is a problem in the analysis of such systems; if a contextual system is examined under two different contexts then a very different set of results may be obtained. This implies that experiments and observations are not merely discovering reality, there is a very real sense in which they might be creating some aspect of that reality. This result has been examined comprehensively for quantum systems where a significant mathematical formalism has been developed exploring its ramifications. Despite this well developed formalism the meaning of quantum theory is yet to be understood, and it will be proposed here that this lack of interpretation is due to the actual complexity of quantum systems.

According to standard quantum theory[1] there are two forms of time evolution exhibited by the wavefunction, $|\psi(\mathbf{x}, t)\rangle$, which represents the current state of a quantum system (in a complex, linear vector space known as a *Hilbert space*):

(i) A continuous linear evolution represented by the Schrödinger equation[2] that occurs in all situations but that of *measurement*, when,

(ii) an instantaneous, nonlinear collapse occurs. After this collapse, the system is found in one of a set of possible states all of which are eigenvectors given by a combination of the measurement apparatus and the system itself. The result of the measurement is probabilistically determined from the associated eigenvalue of the eigenvector.[3]

Because of this process of physical collapse it is not always possible to perform two different experiments upon the same physical system. Consider for example the attempt to measure a given electron's position and momentum. Measuring its position involves a process akin to shining a light (a beam of photons) upon the electron and recording which ones return to a detector, but in doing this we have hit the electron and imparted a certain new momentum to it which means that we can no longer measure the electron's original momentum. Hence, it is not possible to measure both the position and the momentum of the electron, a fact represented by the Heisenberg Uncertainty principle. This principle says that if the mathematical operators representing two different experiments commute, then they might be performed together and still achieve an arbitrary accuracy *i.e.* they are *compatible*, and conversely that they are incompatible if their operators do not commute. Thus, in performing measurements upon quantum systems, we must be careful that we are not performing incompatible experiments; in some cases the context matters. However, the situation is actually worse than this, even commuting observables are sometimes incompatible.

This result has been demonstrated unequivocally in two independent proofs, a discrete one due to

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[1]What follows is an extremely abbreviated account of the quantum formalism, only what is necessary to the current discussion is covered despite the fact that the interpretation of quantum mechanics, and in particular of measurement is a highly contentious issue (Bell, 1987; Laloë, 2001).

[2]The Schrödinger equation, $i\hbar\frac{d}{dt}|\psi(\mathbf{x}, t)\rangle = H|\psi(\mathbf{x}, t)\rangle$, is the dynamical equation of motion for quantum systems. In this equation $H(\mathbf{x})$ is the Hamiltonian, a Hermitian (hence probability conserving) linear operator that can be derived from the Euler–Lagrange equations of motion of the associated 'classical' system (Bohm, 1989).

[3]It is interesting for our current purposes to consider just how this probability is obtained. Generally, the wavefunction $|\psi\rangle$ is written in terms of a set of basis states $\{|\phi_i\rangle\}$, which are chosen such that they correspond well with the variable to be measured. Thus $|\psi\rangle$ is written as a linear supperposition of the basis states, with weight terms c_i, representing the contribution of each basis state to the actual state, $|\psi\rangle = \sum_i c_i|\phi_i\rangle$. The choice of basis states is governed by the observable to be measured; with a good choice we find that $O|\phi_i\rangle = o_i|\phi_i\rangle$ (*i.e.* the superposition is nondegenerate) and that the spectrum of the operator representing the observable to be measured is real (*i.e.* the operator is self-adjoint, or Hermitian, for the choice of basis). For such a situation, the quantity $|c_i|^2$ is the probability that the eigenvalue o_i is observed in a measurement of $\mathbf{O}$ on the state $|\psi\rangle$. Thus, the concept of measuring apparatus, and its current state is incorporated (albeit implicitly) into the quantum formalism (Bell, 1987; Bohm, 1989).

Kochen and Specker, and a continuous argument made by Bell one year earlier (Mermin, 1993). Each of these theorems rely upon showing that when a quantum system is *entangled*[4] there exists a set of observables for which it is impossible to consistently assign an eigenvalue. This is despite the fact that the sets of observables are commuting (Mermin, 1993; Laloë, 2001). Thus, the requirement for commuting sets of observables is not strong enough to allow for a consistent assignment of eigenvalues. A direct result of this characteristic is that it that it is impossible to completely describe a quantum system without reference to an experimental arrangement. In fact, even spatially separated entangled quantum systems cannot be considered as properly isolated, a fact demonstrated theoretically by Bell (1987) in the by now infamous Bell's Theorem, and experimentally by Aspect and coworkers (Greenstein and Zajonc, 1997), and commonly referred to as *nonlocality* in the physics community. This is very interesting to the current discussion, since an entangled quantum system is really just one for which it is impossible to separate the analysis of two or more constituents; entangled systems just exhibit a special form of nonseparable behaviour.

The phenomena of nonlocality and contextuality are not restricted to quantum systems. More than 20 years ago a straightforward example of a classical system which violates Bell's Theorem was constructed by Aerts (1982). This system can be created by taking the two vessels V_1 and V_2 illustrated in figure 3 and connecting them with a tube. If we mistakenly assume that the system represented by combining V_1 with a reference vessel into which we siphon water is separate from any possible experiments we might want to perform on V_2 then we will be sorely mistaken. Consider for example an attempt to measure the depth of the water in V_2; if the water has already been siphoned into the reference vessel then a different result will be obtained. Thus, the subsystems S_1 and S_2 are not separable from one another.

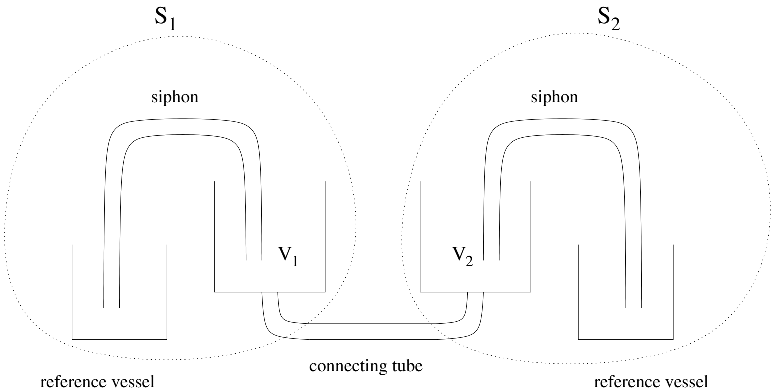

Figure 3. A system consisting of two cubic vessels (which have sides of length 20cm and can each hold 16 litres of water) connected via a tube (which can hold 16 litres) is filled with 32 litres of water. Depending upon the experiments performed upon this system, it can exhibit violations of Bell-type inequalities.

The nonseparability of this experiment can be measured somewhat through the use of a Bell inequality constructed by Aerts for experimental settings of a very general form (see appendix A). The construction of this inequality is left to the appendix, but for now we might ask what characteristics are shared by such a simple classical system and the more traditional entangled states of quantum theory.

The system of figure 3 is entirely classical and it is possible to understand the mechanism that causes its nonseparable behaviour. It is easy to see that it is the tube connecting the two containers. This tube creates a contextual dependency of one vessel upon the other. In the system of figure 3 this dependency is well-understood, but this is not the case in quantum mechanics, and there is no reason to suppose that the mechanism there will be similar. However, we might conclude from this example that it is the contextual dependency between two parts of a quantum system which drives the strange outcomes of quantum mechanics, not the specific quantum mechanism itself. Indeed, a wide variety of novel applications of the quantum formalism are starting to appear; a number of quantum theories of macroscopic systems

[4]An entangled state consists of at least two noninteracting systems A and B, represented by the states $|\psi_A\rangle$ and $|\phi_B\rangle$ on the Hilbert spaces H_A and H_B, the composite state of which is written $|\psi_A\rangle \otimes |\phi_B\rangle$ which cannot itself be separated in a meaningful manner as a product $|\psi_A\rangle|\phi_B\rangle$.

have already been created, often quite successfully (Bruza et al., 2007b). For example, different varieties of the quantum formalism have been applied to situations such as: stock market analysis (Baaquie, 2004); quantum models of the brain (Vitiello, 2001); models of cognitive function and concepts (Aerts and Gabora, 2005; Gabora and Aerts, 2002; Bruza and Cole, 2005); modelling of the process of decision making in situations of ambiguity (Aerts, 2005) *etc.* Thus, the formalism of quantum mechanics appears to be more generally applicable to nonseparable systems than has traditionally been considered the case.

Returning to the containers of water example above, the violation of the Bell inequality derived in appendix A can be used as a test to determine whether the system is exhibiting contextual behaviour or not. This inequality is very general, it does not apply only to the above example. Thus, if a system can be described in a form amenable to the extraction of a Bell-type inequality, then a very straightforward test can be performed to determine whether or not it is exhibiting contextual behaviour. Other general applications of the quantum formalism can be constructed. For example, if a system can be described using a set of basis states that are compatible with the basis states available to an observer then the process by which observation actively influences the state of the system can be modelled as a process of quantum measurement (Bruza et al., 2007a).

Thus, the assumption that our measurements can be performed objectively is not one that can always be made; other experiments, or even the environment of a system can affect the results that we obtain. In particular, objectivity cannot necessarily be assumed in the discussion of nonseparable (*i.e.* contextual) systems. However, there are formalisms already available that might be used to explore this behaviour, specifically, quantum theories appear to have this characteristic. What other methodologies are available for the analysis of systems exhibiting contextual behaviour?

5 New methodologies

> *Contemporary physics does not have the tools to address the problems of innovation, the discovery of patterns, or even the practice of modeling itself, since there are no physical principles that define and dictate how to measure natural structure.* Crutchfield (1994b, p6)

Section 4 has illustrated the way in which two key assumptions of modelling, held almost sacrosanct in the field of science are fast approaching the end of their general validity:

(i) Our models are biased towards objects. While this is a reasonable bias in the modelling of simple systems that can be easily reduced to their constituents, it generally limits our modelling when we attempt to generate complex emergent behaviour.

(ii) The traditional assumption that a system can be cleanly separated from its environment, and that we can perform objective measurements upon some system without in any way impacting upon what is measured is no longer one that can be safely made.

In order to avoid these difficulties, it will be necessary to devise new modelling methodologies. However, the news is not all bad. There are existing techniques that might be used to model high end complexity, but these often entail a significant increase in the difficulty inherent in analysing the model.

5.1 *Relational modelling*

One possible solution to this modelling dilemma is to make use of a more relational approach, shifting the emphasis from objects to the relationships between them, similar to the approach taken in network type theories such as food webs and genetic regulatory networks. However, if we are to avoid the traditional barrier of objects that occurs in the modelling of complex systems then the emphasis must be shifted even more strongly towards the relations between them; without an emphasis upon pre-existing objects it becomes possible to generate new emergent objects (Kitto, 2005, 2006). Such an approach has been successfully applied in the field of fundamental physics, where a simple pregeometric iterative equation appears to generate new emergent behaviour such as three dimensionality (Cahill, 2005; Kitto, 2002, 2006, 2007). The reader is referred to the references for more details.

5.2 *Quantum theories as models of complexity*

Since the quantum formalism is so effective at describing contextual behaviour, another possibility is to make use of this formalism in modelling systems exhibiting high end complexity. In order to achieve this we must carefully re-consider the interpretation of quantum mechanics. Not all of the current interpretations are compatible with such an approach, and indeed a new interpretation presents itself; the quantum formalism may simply be describing contextually dependent complex systems. These issues are considered in depth in a paper that is currently in preparation, but can be briefly summarised as follows. As was discussed in section 4.2, standard quantum theory is capable of modelling the contextual dependencies of systems upon experimental arrangements, and different aspects of the formalism can be used to both test for and to describe contextual behaviour. However, standard quantum theory still only allows for one stable ground state and therefore cannot be used to describe systems that are dynamically evolving and changing. If in addition to contextuality, the generation of new emergent behaviour is required in a model then a shift to Quantum Field Theory (QFT) must be performed (Vitiello, 2001).

The possibility of using QFT to generate emergent behaviour can be illustrated with the following proof of concept first proposed by Kitto (2006). This consists of a dynamical model of the process of sympatric speciation where the original population of a species remains in a single geographical location, but for some reason splits into two distinct subgroups which eventually form two different species.

We define a new term, *Species Equivalent* (SE) to express the fact that the phenotype of any organism belonging to the species is SE to that of any other organism of that species. Species equivalence of a phenotype is defined in a very broad sense for the purposes of the current model. For example, if we consider the phenotypes of two humans we see that they are considered to belong to the one species, and hence are SE, despite the fact that phenotypically they might have different eye, hair and skin colour *etc.* For the purposes of the model, we shall discard this extra individual specific information, $p_{specific}$, and consider just the species relevant portion, p_{SR}, of the phenotype, where the total phenotype is defined as:

$$p_T = (p_{SR}, p_{specific}). \tag{1}$$

Within the one species S, all genotypes give the same SR phenotype,

$$\{g_1, g_2, \dots\} \to p_{SR}, \tag{2}$$

hence there is a sense of symmetry in this system. This can be represented mathematically by incorporating a sense of *niche* into the model. A niche is defined rather broadly in this model as a potential which represents all the relevant environmental factors (or contexts) giving rise to one species. If niche is defined broadly enough then there will only ever be one species in one niche (we will delay a discussion of this idea until the end of this section). Thus, at this point the concepts of genotype, phenotype, species and niche have been naturally defined in a simple symmetry oriented model.

Now, if we represent some ecosystem as a set of fields $p_{SR} = \{p_1, \dots p_N\}$ representing N species equivalent organisms, then we might start to understand sympatric speciation as a process which occurs when an environment, represented by a potential, dynamically changes to a situation of degeneracy. In this case, the description of the system in terms of the field p_{SR} will become inadequate, as dynamical symmetry breaking will have occurred and according to Goldstone's theorem (Kaku, 1993; Marshak, 1993) a number of new fields will emerge, the number of which depends upon the number of broken symmetries. In this case, we might consider the number of broken symmetries to be an indication of the number of ways in which the environment has changed, generating new niches.

A toy model of this phenomenon can be quickly constructed. We write a Lagrangian using the standard Mexican hat potential which describes the interaction of the phenotypic fields with the niche:

$$\mathcal{L} = (\partial_\mu p^* \partial_\mu p) - V(p) \tag{3}$$

where we have written the Lagrangian in terms of the vector field $p = \{g_1, g_2, \dots\}$ which represents the symmetrical species equivalent phenotype in the state space of all possible genotypes, and where[1]

$$V(p^2) = \frac{\mu^2}{2}(p^2) + \frac{1}{4}(p^2)^2. \tag{4}$$

In this potential term, μ^2 is a key indicator of the state of the environment. Assuming that $\mu^2 \to -\mu^2$ under the influence of some environmental factor, the potential will become degenerate and symmetry breaking occur. We are left with a number of new phenotypes equivalent to the number of broken symmetries, or genotypes which do not result in the same p_{SR} as was previously the case. Such a dynamical influence can be easily imagined, consider perhaps a periodic change in sign due to a change in temperature over the course of the seasons, or the steadily decreasing rainfall averages that are arising in some areas due to global warming. Such dynamical influences upon this simple toy system are currently being investigated.

It is interesting to consider some of the implications of this model for our understanding of ecological systems. Firstly, according to this model a niche is defined as a unique set of resource plus locality characteristics associated with some environment that leads to a particular species. In this sense, the term guild (Root, 1967) might perhaps be more accurately used, as this is defined as a group of species that exploits the same class of environmental resources in a similar way. For example the carrion eaters niche is occupied by the Tasmanian devil in Australia, jackals and wild dogs in Africa, and vultures in the Americas. However, as the current purpose of this discussion is to identify a particular set of environmental factors that are utilised by some species in a particular ecosystem, the term niche was chosen. In the current usage, the occupation of a niche is precisely what leads to the identification of a species.

Thus, in this model the concept of niche is definitionally linked with that of species. Instead of the more traditional, object based approach which sees an abiotic landscape and then places organisms upon it, this framework inextricably links species to environment; they can no longer be separated, or decoupled in the model, which is a pleasing result since this is also the case in reality. Thus, genotype, phenotype and environment must be considered holistically within this model. A change in the behaviour of one can have profound consequences for the rest of the system.

Although this modelling process is quite straightforward, it is expected that it will be possible to derive a large variety of models from it, through the utilisation of different time evolutions of the parameter $\mu(t)$, as well as through the use of different fields and interactions represented by the potential term (4). Also, with an understanding of what this model describes, namely the process of differentiation among species, we might start to see a way in which this model might even be extended to other systems. For example, biological development might be understood as a process of cell differentiation coupled with morphogenesis, and the above model can be expected to assist in constructing models of this process.

We must note at this point that the model proposed above is just another formal mathematical model and should not therefore be regarded as *the* model of sympatric speciation or differentiation. As was stated at the beginning of this paper, it is not believed that systems at the high end of the complexity scale can be modelled by one formal or computational model alone, rather, a number of complementary models must be adopted for a full understanding of such systems. This can be seen in the case of the model above, where a complete model of speciation would couple the formal model above with both data and models specific to individual cases of speciation such as, for example, the breaking of a fully-interbreeding population of the butterfly *Maniola jurtina* into two distinct races occupying contiguous geographical areas but unable to interbreed (Ford, 1975). Thus the importance of the above general model of differentiation is that it is indeed capable of generating a more open ended form of emergent behaviour than is generally the case with formal models (Kitto, 2006).

It is likely that quantum field theories have a number of characteristics essential to the modelling of complex emergent behaviour. Some clues for the reason behind this are supplied by early work performed by pioneers such as Primas (1983) who noted that it is possible to perform a perturbation expansion over multiple time scales, by Fröhlich (1988) who proposed that coherent phase correlations will play "a decisive role in the description of biological materials and their activity", and Davydov (1991) who proposed the

[1]Equations (3) and (4) have been straightforwardly taken from a multiple field Mexican hat system often utilised as a simple toy quantum field theory. See for example Kaku (1993).

concept of a biological soliton that resists thermal fluctuations. Taken together these results suggest that QFT has many applications in nontraditional fields, but despite this initial promise there has been a tendency for the larger physics community to ignore these possible extra applications.

As was evident in the above model, the full power of QFT becomes apparent when symmetry breaking is incorporated into the formalism. If a system falls into a situation where its ground state does not have the same continuous symmetry as its dynamics (as represented by its Lagrangian) then it is considered to be exhibiting spontaneous symmetry breaking. According to Goldstone's theorem, a system in such a state will dynamically generate a number of massless bosons, termed Nambu–Goldstone modes (NG-modes), in response to this break in symmetry. The number of NG-modes generated will equal the number of broken symmetries in the system (Marshak, 1993). Due to their massless nature, NG-modes are long-range, they can move through an entire system with no loss of energy, providing long range coherence to such systems (Vitiello, 2001). Because of their boson status many NG-modes may occupy the same ground state without changing the energy of the system. In such systems there can be a number of structurally different ground states, each in a lowest energy configuration. This is not possible in a classical or even a standard (nonrelativistic) quantum system; there is only one lowest energy state in these theories, which means that there is only one way in which such a system can exhibit stable behaviour. On the other hand systems described by a QFT can exist in many different stable configurations, which allows them to change state from one stable ground state to another and in the process generate new behaviour of a very rich form. Because of this phenomenon, QFT is the only quantum formalism capable of generating truly emergent behaviour rather than merely modelling a set of components and their interactions (Umezawa, 1993; Vitiello, 2001; Kitto, 2006). This is to be expected since its basis could be argued to lie in the necessity of modelling the creation and annihilation of particles in modern high energy experiments; QFT was invented in order to model the emergence of new particles.

6 Conclusions

Complexity is a complex concept. It is likely that the identification of a system as complex depends upon both the underlying nature of the system itself and the context in which we wish to understand it. Some systems can be analysed objectively; they can be understood in terms of simpler subcomponents, none of which have contextual responses to environmental conditions. Other systems are not so amenable to such reductive analysis, and this paper has argued that our modelling methodology must be substantially extended before we will be able to fully understand their behaviour. Many of the systems not at present adequately modelled by the scientific method appear to exhibit contextual responses, and in many cases observer driven behaviour.

A contextual system is one that cannot be separated from its surroundings, while a system exhibiting observer driven behaviour will be contextual, but will in addition to this require more than one model for a full description of its behaviour. In contrast, simple systems do not exhibit contextual responses of any kind and can be completely specified by one model. However, the specification of a system as simple itself relies upon an observer who generally ignores a number of irrelevant factors (separates them from the system of interest in formulating a model). For these reasons it is inappropriate to designate a system as either simple or complex, indeed:

> *It must be acknowledged that the way we* look *determines what we* see*, or rather it co-determines the latter, in conjunction with what* there is*.* Kampis (1995, p95)

With a proper understanding of contextuality and observer driven complexity it becomes possible to formulate a scaling of complex behaviour, and to identify the way in which our different models of complexity cope with the inherent contextuality of, not just the systems in which we are interested, but of our very approach to the notion of complexity. We live in a contextual world, how we look can determine what we see, but if we can communicate the way in which this occurs, then we can hope to understand this contextuality, and hence high end complexity.

With an understanding of the scale of complex behaviour it becomes evident that more tools will be required in order to understand those systems at the high end of the scale, and this paper has been a first step in that direction. In particular, the quantum formalism has been proposed as capable of modelling systems exhibiting contextual dependency in general, not just those lying in the microscopic domain. This is due to the ability of the quantum formalism to model both contextual responses to environmental perturbations, and in the case of QFT, to provide a general framework from which contextually emergent behaviour can be generated. Thus, the news is not all bad. While there are a wide number of systems exhibiting contextual, and even observer driven behaviour, they need not be considered subjective. With an increased research effort at the high end of the complexity scale we might hope that more novel analytical tools will be developed and eventually a more complete understanding of the fascinating systems that can be found there.

7 Acknowledgements

The author would like to acknowledge the support, input and guidance of Reg Cahill and Daniel Kortschak. Without their assistance this work would not have been possible.

Appendix A: A classical experiment that violates a Bell inequality — Proof

We start by assuming that four yes-no experiments $\alpha, \beta, \gamma, \delta$ can be performed upon some system of interest, and associating with each experiment μ a variable signifying its outcome X_μ:

$$X_\mu = \begin{cases} +1 \text{ if a yes answer is returned,} \\ -1 \text{ if a no answer is returned.} \end{cases} \tag{A1}$$

It may be possible to combine experiments, obtaining answers to both experiments at the same time. In this case coincidental outcomes of two experiments, $X_{\mu\nu}$, can be defined as follows:

$$X_{\mu\nu} = \begin{cases} +1 \text{ if the answers are correlated, } i.e. \text{ if they are of the form, \{yes,yes\} or \{no,no\}} \\ -1 \text{ if the answers are anticorrelated } i.e. \text{ if they are of the form, \{yes,no\} or \{no,yes\}.} \end{cases} \tag{A2}$$

It is important to note that the coincidence experiment $X_{\mu\nu}$ is a new experiment and must be considered separately from the two individual experiments X_μ and X_ν; there is no reason to assume that there will be a correlation between $X_{\mu\nu}$ and some legitimate combination of the experiments, X_μ followed by X_ν, say.

In order to construct a Bell inequality, Aerts supposes that it is possible to perform compatible coincidence experiments $\alpha\beta$, $\alpha\gamma$, $\delta\beta$ and $\delta\gamma$ on a system consisting of two parts S_1 and S_2 which are localised in two different regions of space. Experiments α and δ are performed on S_1 and experiments β and γ are performed on S_2 (see figure 3). Assuming that the system is separable (or noncontextal) involves the adoption of the relations:

$$X_{\alpha\beta} = X_\alpha X_\beta,\ X_{\alpha\gamma} = X_\alpha X_\gamma,\ X_{\gamma\beta} = X_\gamma X_\beta \text{ and } X_{\delta\gamma} = X_\delta X_\gamma. \tag{A3}$$

Which amounts to saying something like 'the trickier coincidence experiment $X_{\alpha\beta}$ need not be performed, it is possible to perform X_α, reset the apparatus and then perform X_β, finally adding the results in some well defined manner to obtain the result that *would have* been obtained had the coincidence experiment been performed.' This assumption is analogous to the assumption made by Bell when constructing the original Bell inequality (Bell, 1964), and we can make use of his result to construct an inequality of the following form (Aerts, 1982):

$$|X_{\alpha\beta} - X_{\alpha\gamma}| + |X_{\delta\beta} + X_{\delta\gamma}| \leq 2. \tag{A4}$$

Which must be satisfied if the coincidence experiments are indeed separable. Thus, we have a mathematical identity which can be used to test the separability of any experimental arrangement which has this form.

Aerts then chooses the experimental arrangement illustrated in figure 3, and lists four specific experiments that can be performed upon this apparatus as follows,

α: tests whether the volume of water contained in the vessels is more than 10 litres. The experiment is performed by siphoning off water from one of the vessels into a reference vessel with volume of 10 litres. If the reference vessel overflows then we answer yes, and if it does not overflow by the time that the first vessel is empty then we answer no.

β: tests whether the depth of the water in the vessel is more than 15cm. The experiment is performed by placing a ruler vertically in the vessel, and reading off the height of the water on the ruler. If the water is higher than 15cm then we answer yes, if not then we answer no.

γ: tests whether the water is drinkable. The experiment is performed by taking a spoonful of water from the vessel and drinking it. After five minutes, if we are feeling good then we answer yes, if we are ill then we answer no.

δ: tests whether the water is transparent. The experiment is performed by taking a spoonful of water and placing it in a glass which is then held against a light source. If the light gets through the water then we answer yes, if not then we answer no.

The coincidence experiment $X_{\alpha\beta}$ returns a value -1, as the two experiments are anticorrelated. That is, the siphoning of the water into the reference vessel for the purpose of experiment α drops the water level in both V_1 and V_2 by the same amount due to the tube connecting them. From the geometry of the vessels (see figure 3), we ascertain that if there is more than 10 litres in V_1 then there must be less than 15cm of water in each of the vessels when we measure the depth of the water; if experiment α returns a yes then β must return a no. In a similar manner we can find situations where

(i) $X_{\alpha\gamma} = +1$. That is, more than 10 litres of water gets emptied into the reference vessel, and the water is drinkable.

(ii) $X_{\delta\beta} = +1$. That is, taking a spoonful of water to determine the transparency of the water (which we take to be transparent) does not lead to the depth of the water changing to less than 15cm.

(iii) $X_{\delta\gamma} = +1$. That is, the water is both transparent and drinkable.

These outcomes are then combined giving:

$$|X_{\alpha\beta} - X_{\alpha\gamma}| + |X_{\delta\beta} + X_{\delta\gamma}| = +4, \tag{A5}$$

which is in clear violation of equation (A4). Thus, this system is nonseparable, and the assumptions (A3) are invalid. We see that if a system assumed to be separable in the above manner violates an inequality of similar form to (A4) then our assumption is invalid. Nonreductive techniques must be utilised in order to generate a full understanding of the system.

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